Decomposition Algorithm Development

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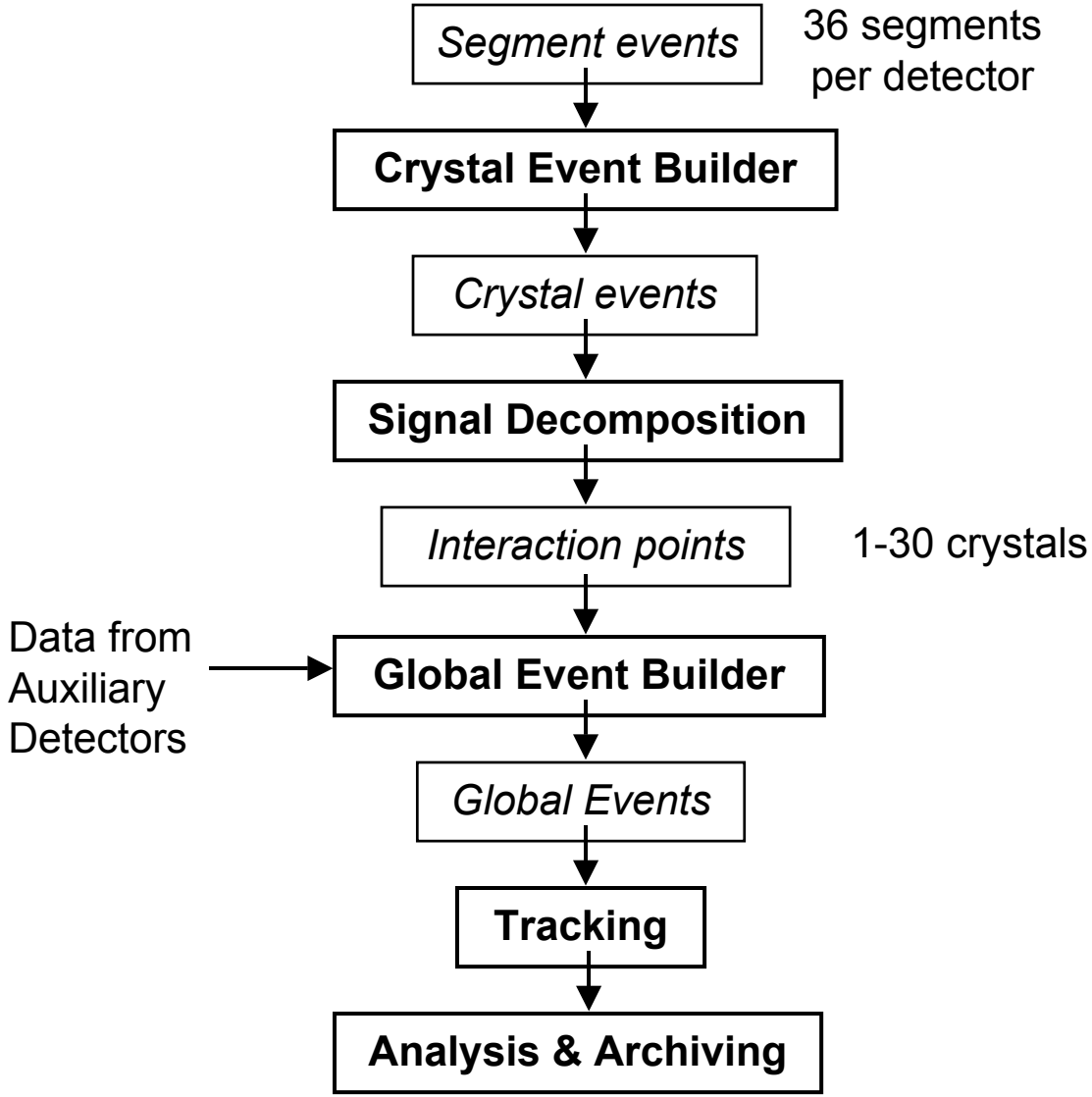
Event Processing

Event Building
Data Flow:



Signal Decomposition

Digital pulse processing of segment data

- Uses data from both hit segments and image charges
- Extracts multiple interaction positions & energies
- Uses a set of calculated basis pulse shapes
- Ideally suited to parallel processing

Algorithms:

- Adaptive Grid Search
- Constrained Least-Squares / Sequential Quadratic Programming
- Singular Value Decomposition

Requires about 90% of total CPU cycles
- the major processing bottleneck

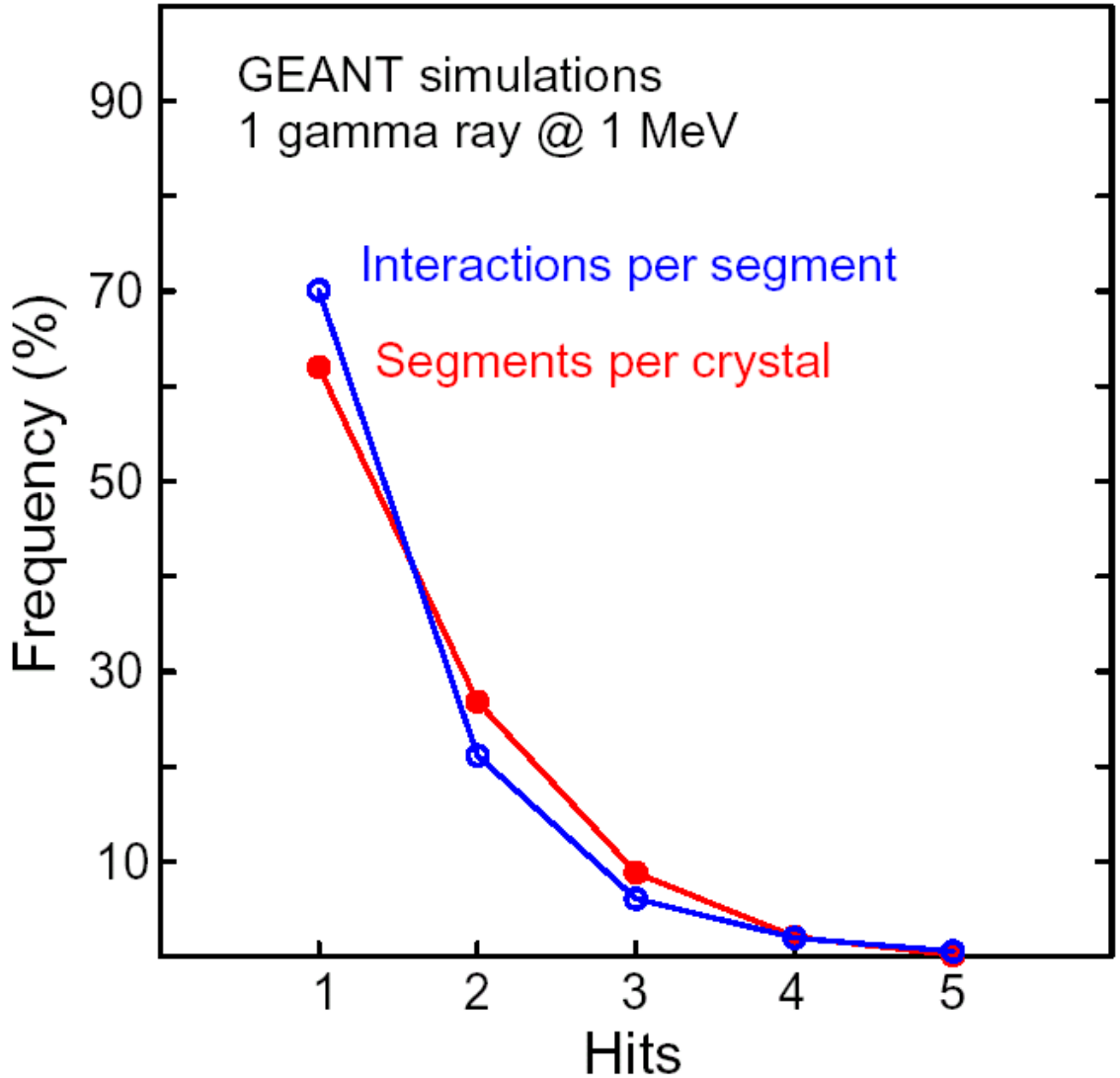
Signal Decomposition

Most hit crystals have one or two hit segments

Most hit segments have one or two interactions

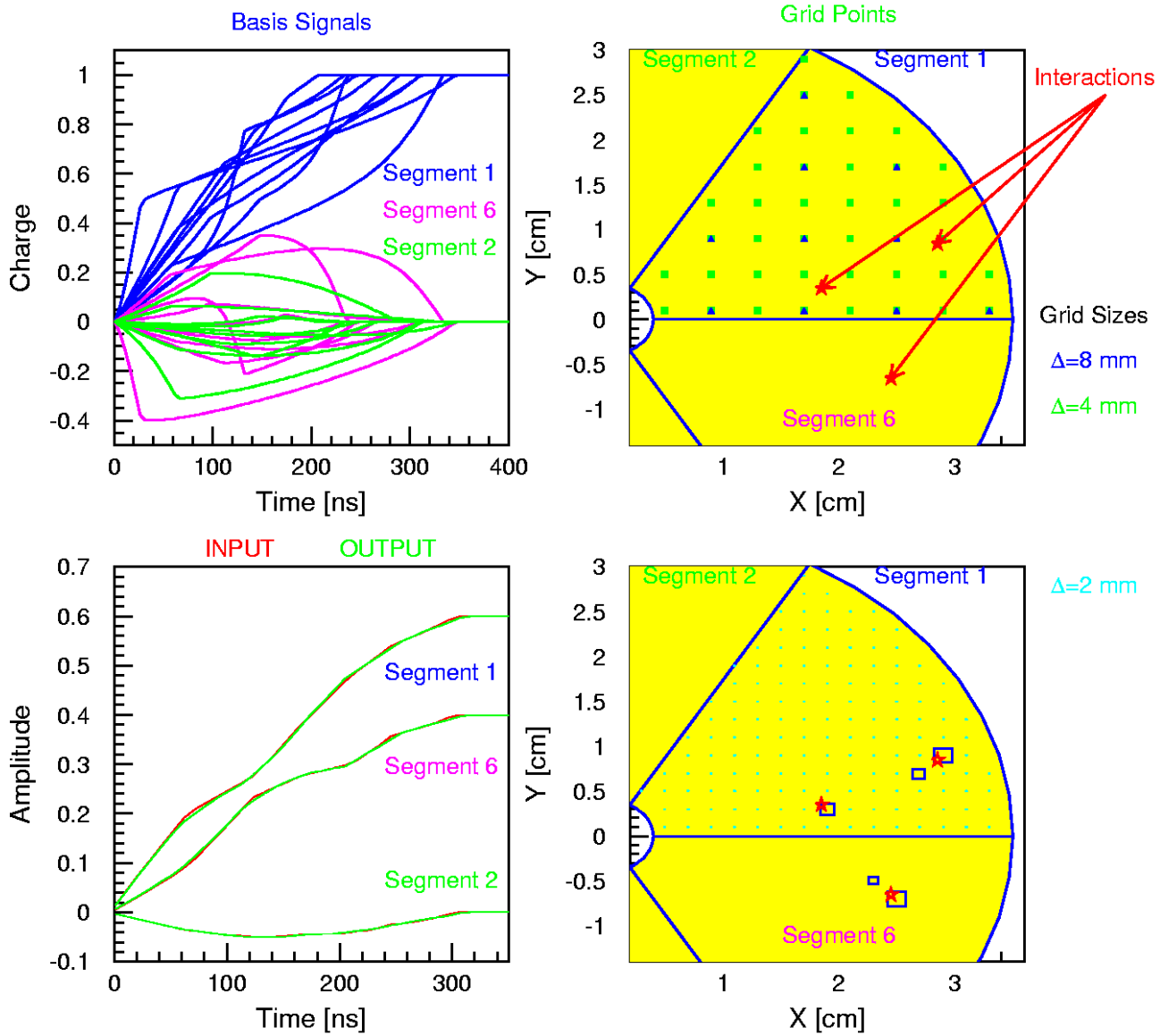
CPU time goes as
- AGS : ~ 100^n
- SVD : ~ n^2
- SQP : ~ n

for n interactions



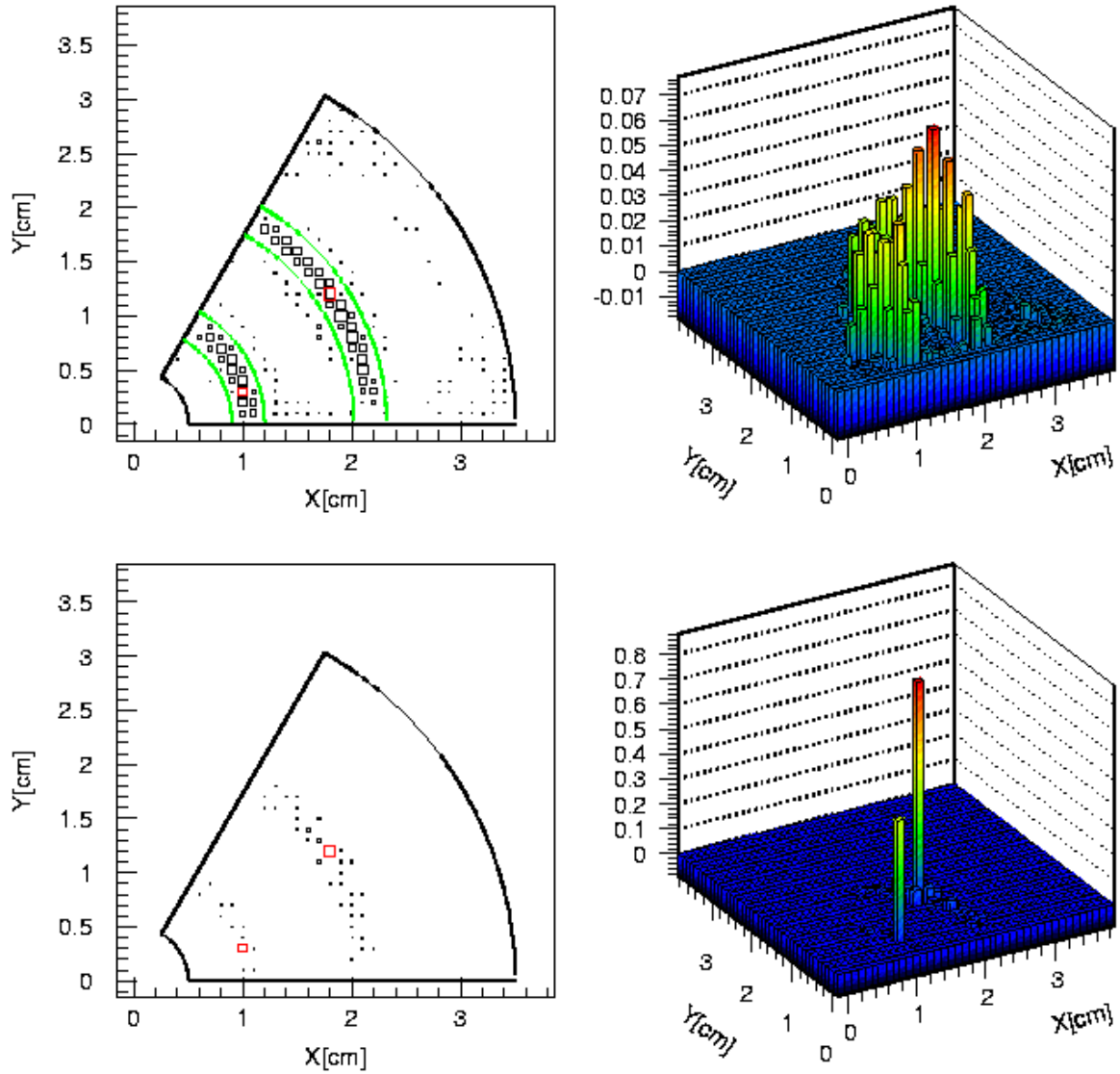
Signal Decomposition

Adaptive Grid Search algorithm:



Signal Decomposition

Singular Value Decomposition algorithm:



Results of SVD in three dimensions for two interactions indicated by red squares. The upper row shows obtained probability distributions taking all possible positions into account. The bottom row shows the results after limiting the radius to the regions between the green lines. The final result agrees well with the input positions.

Signal Decomposition

Decomposition Algorithm – Recent Work

- Adaptive Grid Search,
  followed by constrained least-squares
- Grid search in position only; energy fractions are fitted
  (see next two slides)
- Assumes 1 or 2 interactions per hit segment
- Proven to give <u>excellent</u> results for *single-segment* events
  - Converges for 100% of events
  - Reproduces positions of simulated events to < ½ mm
  - Very fast; ~ 5 ms/event/CPU

Signal Decomposition

Linear Least-Squares

For two interactions of energies e_i, e_j at locations i and j, the calculated signal is $C_{kt} = (e_i s_{ikt} + e_j s_{jkt})$ where k is the segment and t the time step. s_{ikt} is the basis signal calculated at point i.

If the observed signal is O_{kt}

$$\chi^2 = \sum_{kt} \frac{(O_{kt} - C_{kt})^2}{\sigma_{kt}^2} = \frac{\sum_{kt}(O_{kt} - e_i s_{ikt} - e_j s_{jkt})^2}{\sigma^2} \qquad (1)$$

where $\sigma_{kt} = \sigma$ is the uncertainty (noise) in O_{kt}, assumed independent of k, t.

We want a minimum in χ^2, *i.e.*

$$\frac{\partial \chi^2}{\partial e_i} = \frac{\partial \chi^2}{\partial e_i} = 0 \qquad (2)$$

$$\frac{\partial \chi^2}{\partial e_i} = \frac{2\sum_{kt}(O_{kt} s_{ikt} - e_i s_{ikt}^2 - e_j s_{ikt} s_{jkt})}{\sigma^2} = 0 \qquad (3)$$

Signal Decomposition

Thus we get two equations in two unknowns:

$$\sum_{kt} O_{kt} s_{ikt} - e_i \sum_{kt} s_{ikt}^2 - e_j \sum_{kt} s_{ikt} s_{jkt} = 0 \tag{4}$$

$$\sum_{kt} O_{kt} s_{jkt} - e_j \sum_{kt} s_{jkt}^2 - e_i \sum_{kt} s_{ikt} s_{jkt} = 0 \tag{5}$$

We can *precalculate*

$$\sum_{kt} s_{ikt}^2$$

and

$$\sum_{kt} s_{ikt} s_{jkt}$$

once for all events, and

$$\sum_{kt} O_{kt} s_{jkt}$$

once per event.

Signal Decomposition

<u>Some numbers</u> for adaptive grid search:

~35000 grid points in 1/6 crystal (one column, 1x1x1 mm)

2x2x2mm (slices 1-3) or 3x3x3 mm (slices 4-6) coarse grid gives
$N \leq 600$ course grid points *per segment.*

For two interactions in one segment, have $N(N-1)/2 \leq 1.8 \times 10^5$ pairs of points for grid search. This takes ~ 3 ms/cpu to run through.

But $(N(N-1)/2)^2 \sim 3.2 \times 10^{10}$ combinations for two interactions in each of 2 segments; totally unfeasible!

Limit N to only 64 points; then $(N(N-1)/2)^2 \sim 4 \times 10^6$
-- this may be okay. But 4 unknowns will require matrix inversion.

But $(N(N-1)/2)^3 \sim 8 \times 10^9$ combinations for two interactions in each of 3 segments; still impossible.

Signal Decomposition

Adaptive grid search fitting:

Energies e_i and e_j are constrained, such that $0.1(e_i+e_j) \leq e_i \leq 0.9(e_i+e_j)$

Once the best pair of positions (lowest χ^2) is found, then all neighbor pairs are examined on the finer (1x1x1 mm) grid. This is 9x9 = 81 pairs. If any of them are better, the procedure is repeated.

For this later procedure, the summed signal-products cannot be precalculated.

Finally, nonlinear least-squares (SQP) can be used to interpolate off the grid. This improves the fit ~ 50% of the time.

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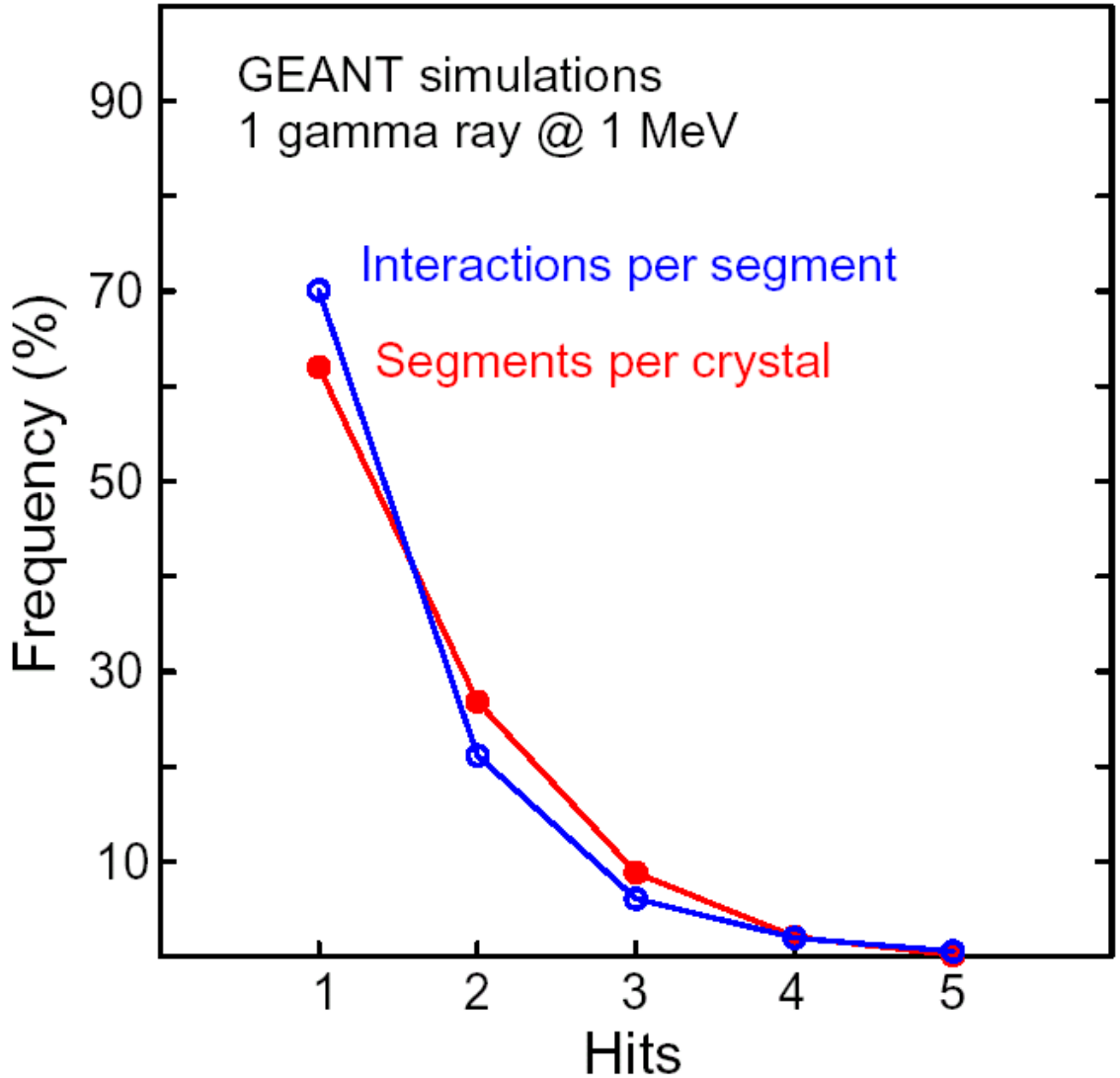
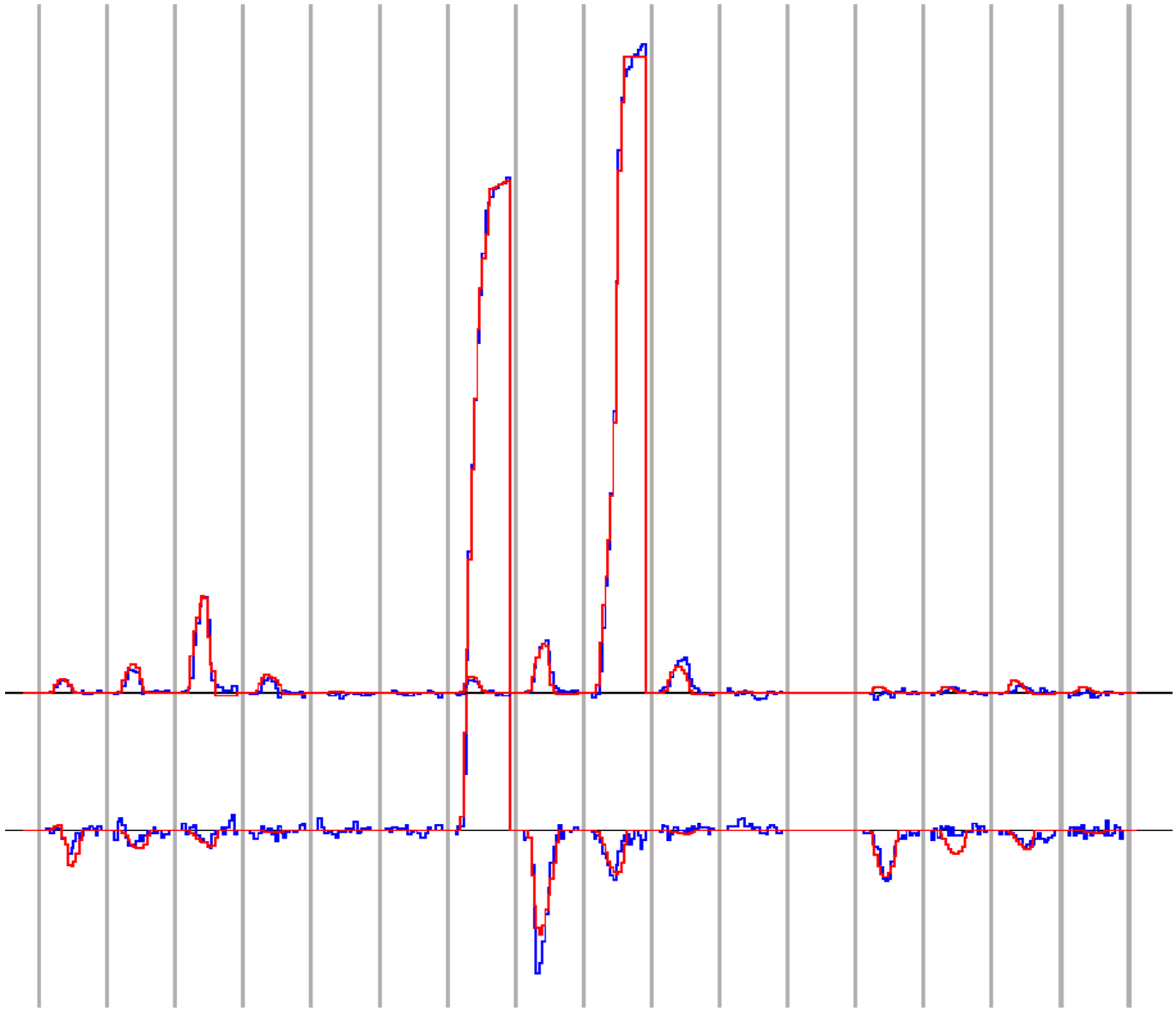


Signal Decomposition

Example events

Blue: measured

Red: fitted



Signal Decomposition

Decomposition Algorithm: To-Do List

- Improve and speed up *multi-segment* algorithm
- Allow more interactions per segment?
- Explore Singular Value Decomposition:
  SVD $\rightarrow$ grid search $\rightarrow$ least-squares
- Replace Cartesian coordinate system with
  cylindrical or quasi-cylindrical coordinate system
- Deal with irregular-hexagonal detectors
- Examine failure modes in detail

Some will be done in Phase-I SBIR with Tech-X Corp.